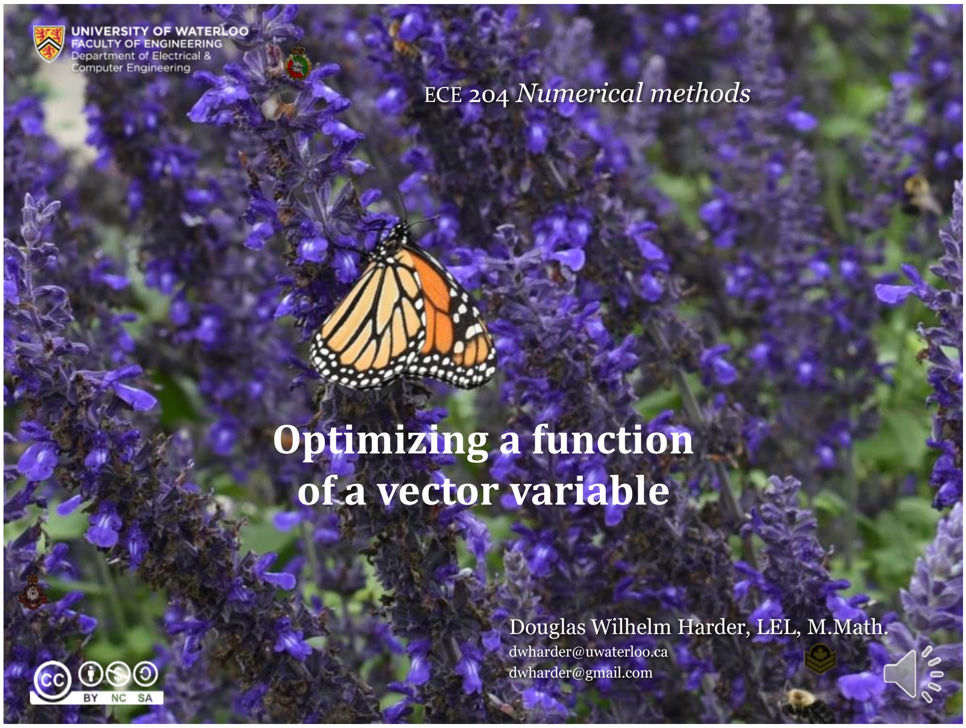


1

Introduction

- In this topic, we will
  - Describe what will be covered in the next section
    - Optimization of functions of a vector variable
  - Discuss how this is more difficult than finding a minimum of a function of a real variable
  - Describe the algorithms that will be covered

2

2

The problem

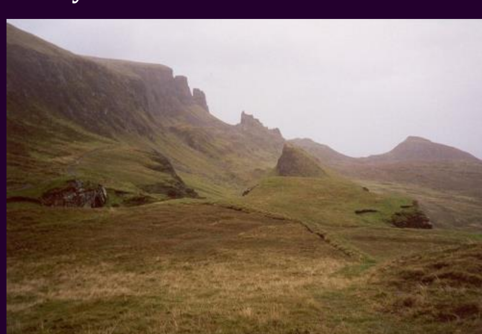
- With a differentiable function of one variable,
  if the function increases in one direction, it decreases in the other
- With a function of even a 2-dimensional vector,
  we now have infinitely many directions to consider
  - If the function is sufficiently differentiable,
    we can calculate the gradient
    - The gradient yields the direction of maximum increase
    - The opposite direction is the direction of maximum decrease

3

3

Getting home

- Suppose you're in the Scottish highlands and its foggy
  - You'd like to get home, which is in the valley
- Given no global information, to get down in the most timely,
  you'd walk in the direction that is steepest downhill



Warwick Bradly August 2001 on Canon Elph

4

4

Properties of an extrumum

- For a sufficiently differentiable function at a minimum, the slope in any direction is zero
  - Unfortunately, the function at a point could be a minimum in every variable but one, causing the point to be a saddle point
  - For example, this function has a minimum at **u** = **0**:

$$f(\mathbf{u}) = u_1^2 + u_2^2$$

  - Both of these are saddle points at that same point:

$$f(\mathbf{u}) = u_1^2 + u_2^3$$

$$f(\mathbf{u}) = u_1^2 - u_2^2$$

Looking ahead

- We will look at:
  - The Hooke-Jeeves method
    - A generalization of the step-by-step algorithm that walks towards the minimum
  - Newton's method for finding extrema in n dimensions
    - A generalization of Newton's method applied to the derivative
  - Gradient descent
    - Converting a problem in n dimensions to a problem in one dimension

Summary

- Following this topic, you now
  - Have an overview of the ideas to be covered in this section
  - Understand that we will look at three algorithms for optimizing a real-valued function of a vector variable

Optimizing a function of a vector variable

References

[1] https://en.wikipedia.org/wiki/Mathematical_optimization

Acknowledgments

None so far.

Colophon

These slides were prepared using the Cambria typeface. Mathematical equations use Times New Roman, and source code is presented using `Consolas`. Mathematical equations are prepared in MathType by Design Science, Inc.
Examples may be formulated and checked using Maple by Maplesoft, Inc.

The photographs of flowers and a monarch butter appearing on the title slide and accenting the top of each other slide were taken at the Royal Botanical Gardens in October of 2017 by Douglas Wilhelm Harder. Please see

https://www.rbg.ca/

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